FRIDAY TALKS @ TUE.AI #300126

THE VALUE OF AMBIGUOUS COMMITMENT WITH MULTIPLE FOLLOWERS

THIS IS JOINT WORK WITH...

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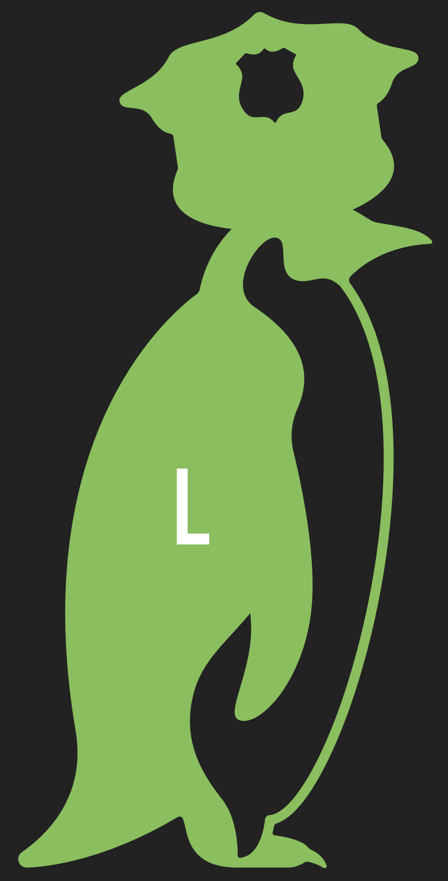
SIMPLE STACKELBERG GAME



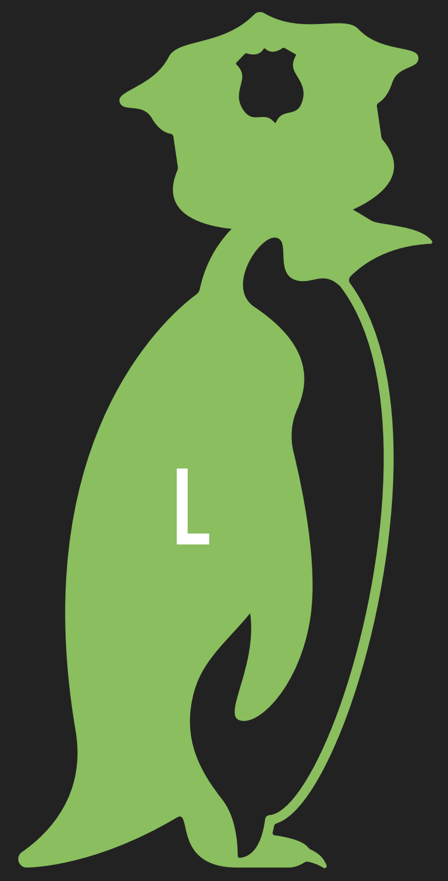
$$\mathscr{A}_L := \{a, b, c\}$$

L	a	b	c
i	u(a,i)	u(b,i)	u(c,i)
ii	u(a,ii)	u(b,ii)	u(c,ii)

$$\mathscr{A}_F := \{i, ii\}$$

F	a	b	c
i	v(a,i)	v(b,i)	v(c,i)
ii	v(a,ii)	v(b,ii)	v(c,ii)

SIMPLE STACKELBERG GAME



$\downarrow p_L$

L	a	b	c
i	u(a,i)	u(b,i)	u(c,i)
ii	u(a,ii)	u(b,ii)	u(c,ii)

F	a	b	c
i	v(a,i)	v(b,i)	v(c,i)
ii	v(a,ii)	v(b,ii)	v(c,ii)

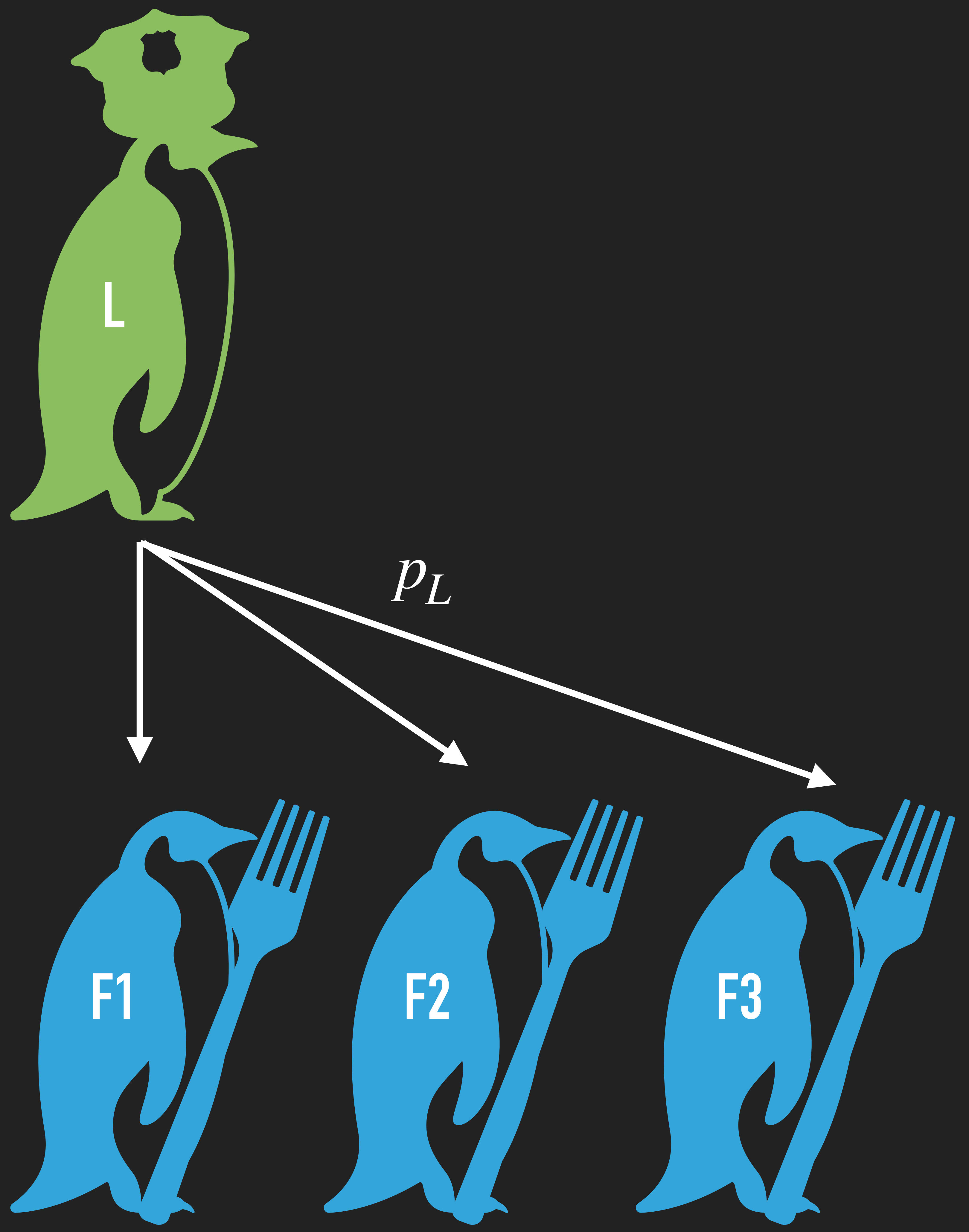
1. Leader chooses $p_L \in \Delta(\mathscr{A}_L)$
2. Follower best responds $\mathrm{BR}(p_L) = \arg\max_{a \in \mathscr{A}_F} V(p_L, a)$ *
3. Leader obtains the Stackelberg value $U(p_L, \mathrm{BR}(p_L))$**

* $V(p, q) = \mathbb{E}_{a_L \sim p, a_F \sim q}[v(a_L, a_F)]$

** $U(p, q) = \mathbb{E}_{a_L \sim p, a_F \sim q}[u(a_L, a_F)]$

COUPLED STACKELBERG GAME

$\forall F \in \mathscr{F}, u_F : \mathscr{A}_L \times \mathscr{A}_F \to \mathbb{R}$



$\forall F \in \mathscr{F}, \quad v_F : \mathscr{A}_L \times \mathscr{A}_F \to \mathbb{R}$

1. Leader chooses $p_L \in \Delta(\mathscr{A}_L)$
2. All followers best respond

   $$\mathrm{BR}_F(p_L) = \arg\max_{a \in \mathscr{A}_F} V_F(p_L, a), \forall F \in \mathscr{F}$$

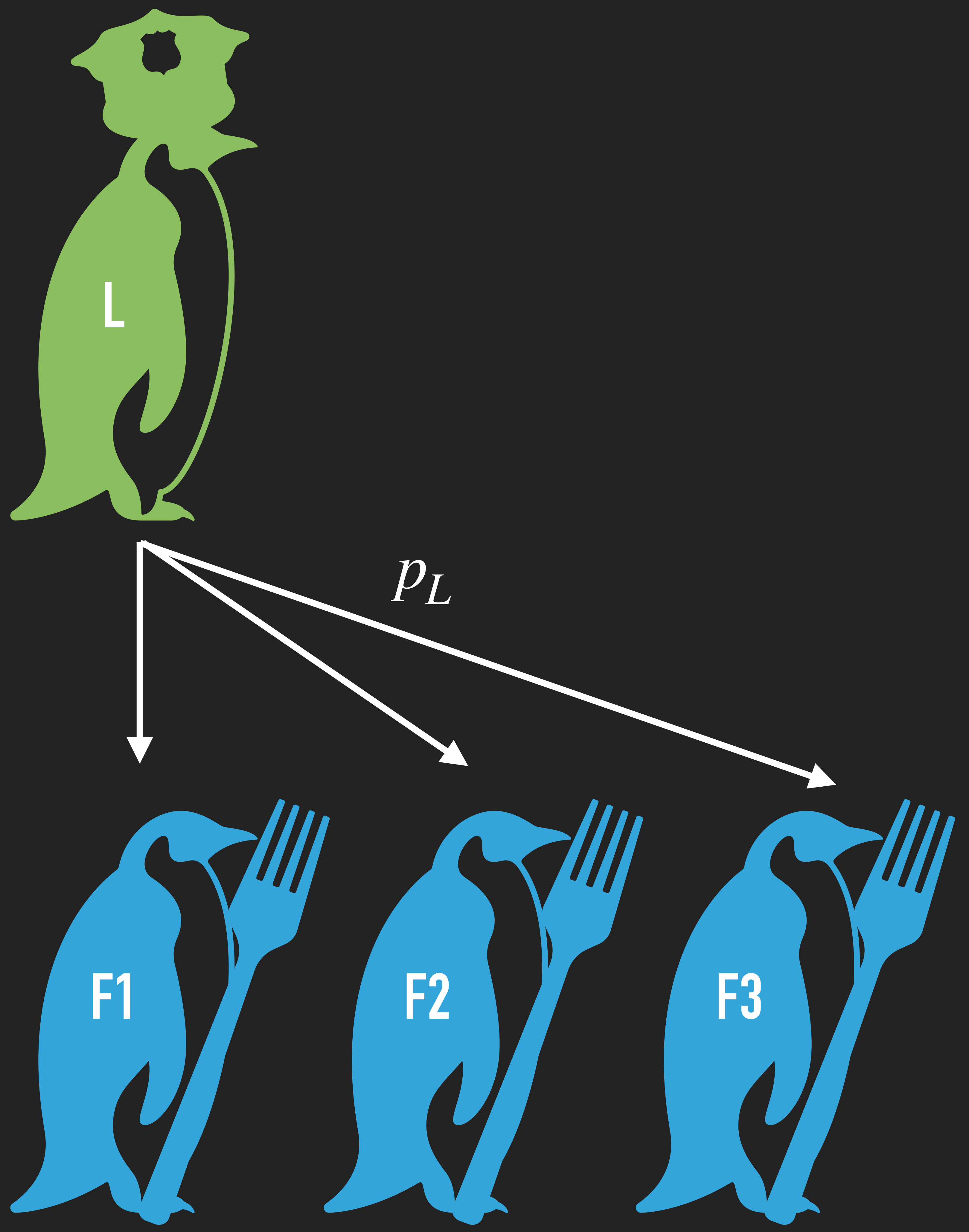
3. Leader obtains the Stackelberg value

   $$\sum_{F \in \mathscr{F}} U_F(p_L, \mathrm{BR}_F(p_L))$$

AMBIGUOUS COMMITMENT

COUPLED STACKELBERG GAME WITH AMBIGUOUS COMMITMENT

$\forall F \in \mathscr{F}, u_F : \mathscr{A}_L \times \mathscr{A}_F \to \mathbb{R}$



$\forall F \in \mathscr{F}, \quad v_F : \mathscr{A}_L \times \mathscr{A}_F \to \mathbb{R}$

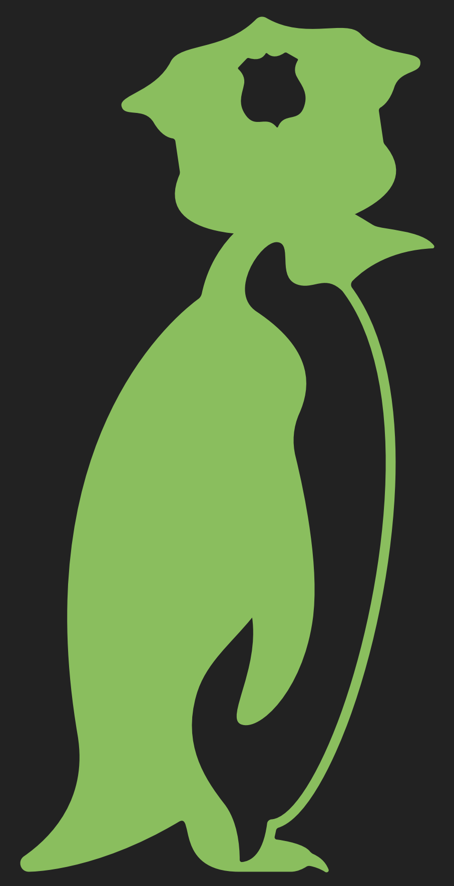
1. Leader chooses $P_L \subseteq \Delta(\mathscr{A}_L)$, closed cvx.
2. All followers best respond
$$\mathrm{BR}_F(P_L) = \arg \max_{p \in \Delta(\mathscr{A}_F)} \min_{p_L \in P_L} V_F(p_L, a), \forall F \in \mathscr{F}$$
3. Leader obtains the Stackelberg value
$$\min_{p_L \in P_L} \sum_{F \in \mathscr{F}} U_F(p_L, \mathrm{BR}_F(P_L))$$

THEOREM I (INFORMAL)

If there is only a single follower, the best classical commitment achieves an equal or higher Stackelberg value than any ambiguous commitment does.

BUT

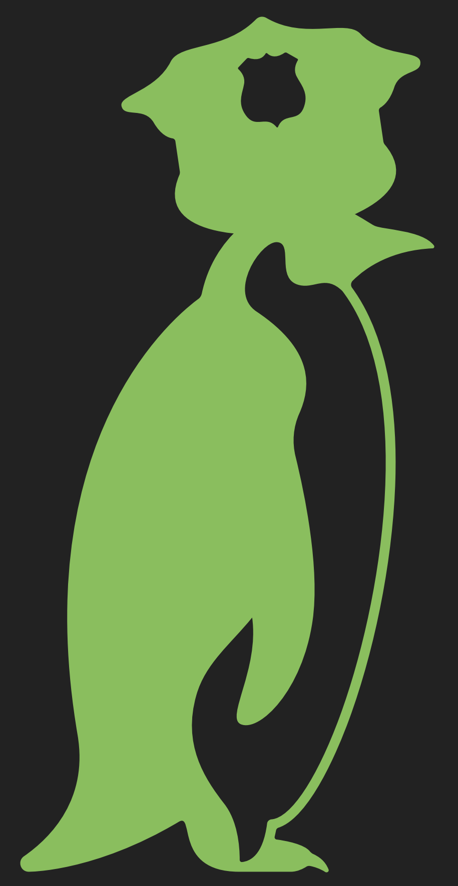
How about a coupled Stackelberg game with multiple followers?



A	A B	A	B	×
	1	1	1	1
	0	0	4	4

B	A B	A	B	×
	1	1	1	1
	0	4	0	4

Zero-sum game with two followers (followers maximize, leader minimizes)

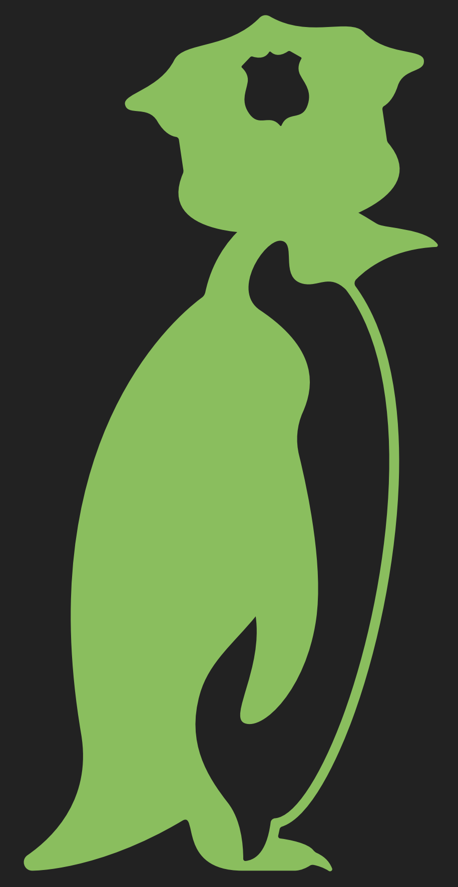


A	A B	A	B	
	1	1	1	1
	0	0	4	4

RESOURCE-CONSTRAINT

B	A B	A	B	
	1	1	1	1
	0	4	0	4

Zero-sum game with two followers (followers maximize, leader minimizes)



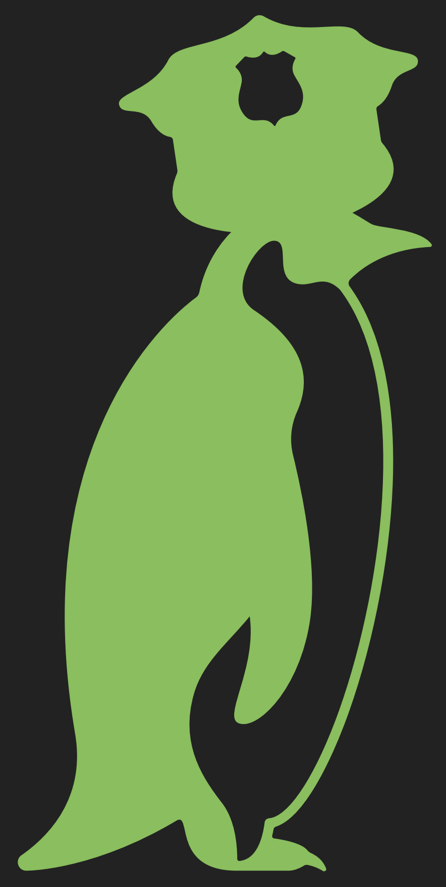
Zero-sum game with
two followers
(followers maximize,
leader minimizes)

A	B	A
	1	1
	4	0

B	B	A
	1	1
	0	4

A SECOND FARM CONTROL GAME

EVOCATION OF RESPONSE PATTERN UNREACHABLE BY CLASSICAL COMMITMENT



Zero-sum game with two followers (followers maximize, leader minimizes)

A	B	A
	1	1
	4	0

B	B	A
	1	1
	0	4

THEOREM II (INFORMAL)

There exists a two follower zero-sum game in which ambiguous commitment leads to an arbitrarily higher Stackelberg value than any classical commitment.

1. A poly-runtime algorithm for finding the optimal ambiguous commitment in a 2xN-Stackelberg games with M followers.
2. NP-Hardness of finding the optimal ambiguous commitment in KxN-Stackelberg games with a fixed number of followers.

"AMBIGUOUS COMMITMENTS HELP TO RAISE THE STACKELBERG VALUE IN MULTIPLE FOLLOWER GAMES."

— WINSTON CHURCHILL … OR MAYBE SOMEONE ELSE